

Precalc Warm Up # 8-1

Mike takes 4 hours longer than Trent to complete a job. If they work together they can complete the job in 10 hours. How long does it take each of them working alone? Identify variables, set up the equation, then solve.

Let t = time it takes Trent to complete the job alone.
 $t + 4$ = time for Mike alone ≈ 22 hrs

Factorable? discriminant
 $2 - 4ac$
 $256 - 4(1)(-40)$
 $256 + 160$
 416

$$\frac{(t+4)}{(t+4)} \cdot \frac{1(10)}{t} + \frac{1(10)}{t+4} \cdot \frac{t}{t} = 1$$

LCD: $t(t+4)$

$$\frac{10t + 40 + 10t}{t(t+4)} = 1$$

cross products

$$t^2 + 4t = 20t + 40$$

$$t^2 - 16t - 40 = 0$$

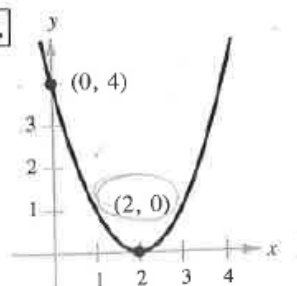
$$t \approx 18 \text{ hrs.}$$

Use quad. form.
 $t = \frac{16 \pm \sqrt{416}}{2(1)}$

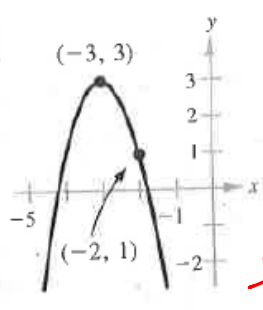
HW Questions: p. 180

In Exercises 7–12, find an equation of the parabola.

7.



11.



$$y = a(x-h)^2 + k$$
$$y = a(x+3)^2 + 3$$

plug in $(-2, 1)$
for x & y to find a

In Exercises 13–30, sketch the graph of the given quadratic function. Identify the vertex and intercepts.

17. $f(x) = (x + 5)^2 - 6$

19. $h(x) = x^2 - 8x + 16$

23. $f(x) = x^2 - x + \frac{5}{4}$

27. $h(x) = 4x^2 - 4x + 21$

In Exercises 31–34, find the quadratic function that has the indicated vertex and whose graph passes through the given point.

33. Vertex (5, 12) Point: (7, 15)

$$y = a(x-h)^2 + k$$

$$y = a(x-5)^2 + 12$$

In Exercises 35–40, find two quadratic functions whose graphs have the given x-intercepts. (One function has a graph that opens upward and the other has a graph that opens downward.)

37. (0, 0), (10, 0) $\rightarrow f(x) = x(x-10)$ opens up
 $f(x) = -x(x-10)$ opens down.

Find two positive real numbers where..

43. The sum of the first and twice the second is 24 and the product is a maximum. *outcome*

let f = first number
 n = 2nd number

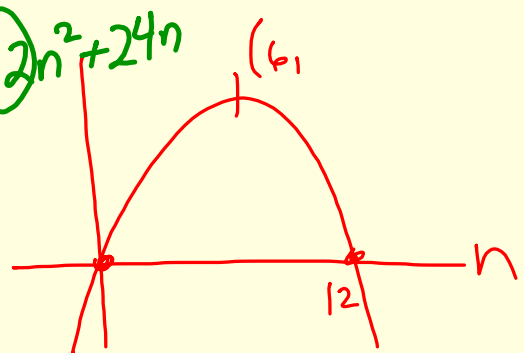
$$f + 2n = 24$$

$$f = 24 - 2n$$

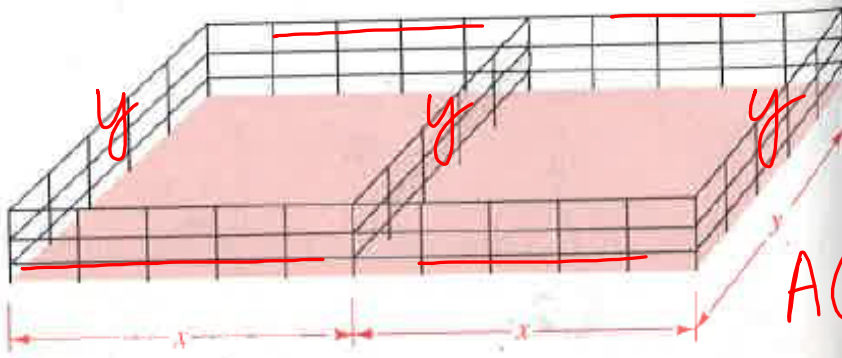
product = $f \cdot n$

$$g(n) = (24 - 2n)(n)$$

$$g(n) = -2n^2 + 24n$$



47. A rancher has 200 feet of fencing to enclose two adjacent rectangular corrals (see figure). What dimensions will produce a maximum enclosed area?

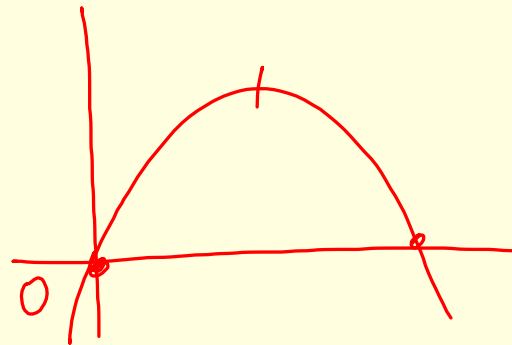


$$A = 2xy$$

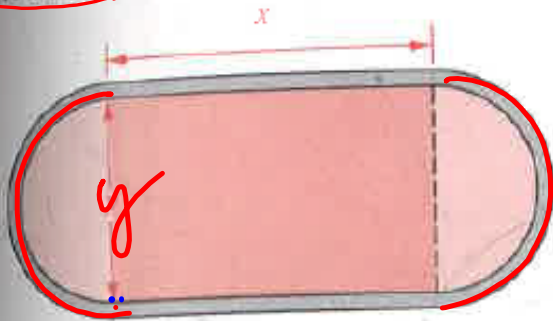
$$A(x) = 2x \left(\frac{200 - 4x}{3} \right)$$

$$4x + 3y = 200$$

$$\frac{3y}{3} = \frac{200 - 4x}{3}$$



8. An indoor physical fitness room consists of a rectangular region with a semicircle on each end (see figure). The perimeter of the room is to be a 200-meter running track. What dimensions will produce a maximum area of the rectangle?



$$C = \pi d$$

$$A = xy$$

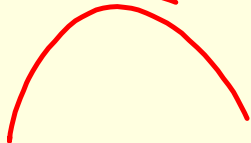
Perimeter

$$2x + \pi y = 200$$

Solve for y , then plug in

Max at the vertex ...

$$A(x) = x \left(\frac{200 - 2x}{\pi} \right)$$



51. Let x be the amount (in hundreds of dollars) a company spends on advertising, and let P be the profit, where

$$P = 230 + 20x - 0.5x^2$$

Axis of symmetry $-\frac{b}{2a}$

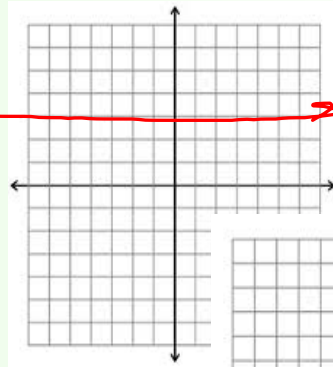
What expenditure for advertising gives the maximum profit?

What x

Graphing 0, 1, and 2 degree polynomials:

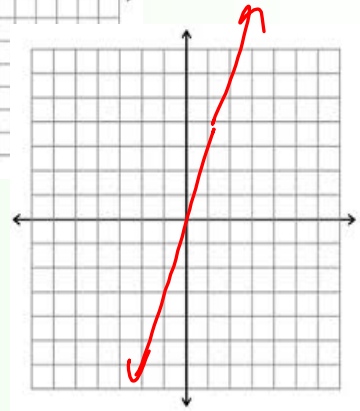
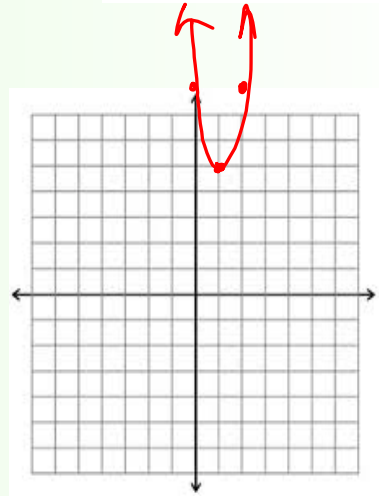
Degree 0

Ex: $f(x) = 3x^0$



Degree 1

Ex: $f(x) = 3x$

Degree 2 Ex: $f(x) = 3(x-1)^2 + 5$ 

Higher degree polynomials

Odd power functions.

$$f(x) = x^3 \quad \begin{array}{c|c|c|c|c|c|c} x & -2 & -1 & 0 & 1 & 2 & \frac{1}{2} \\ \hline y & -8 & -1 & 0 & 1 & 8 & \frac{1}{8} \end{array}$$

$$f(x) = (x+2)^3 + 1 \quad \leftarrow \text{up 1}$$

\swarrow
left 2

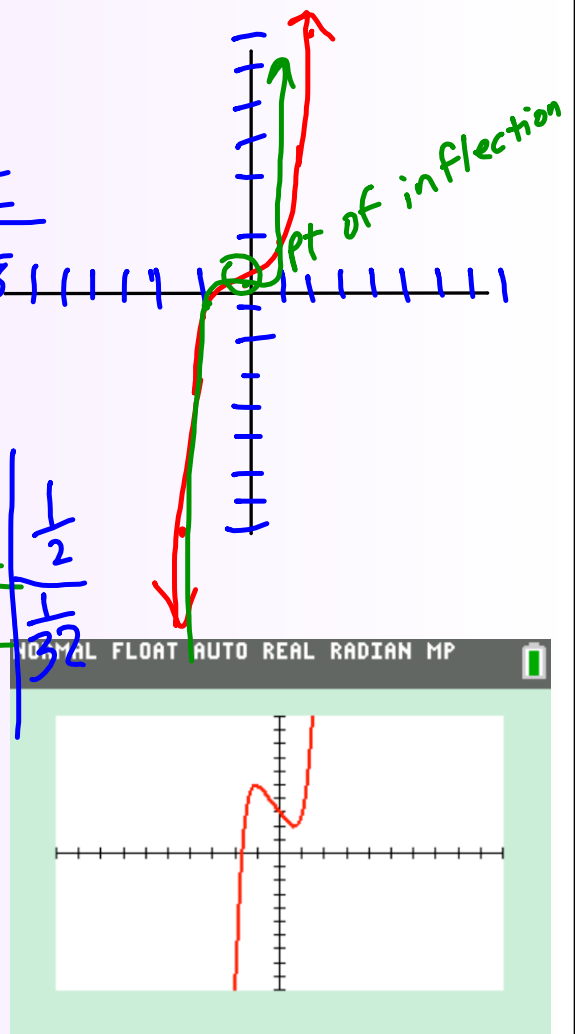
$$f(x) = x^5 \quad \begin{array}{c|c|c|c|c|c|c} x & -2 & -1 & 0 & 1 & 2 & \frac{1}{2} \\ \hline y & -32 & -1 & 0 & 1 & 32 & \frac{1}{32} \end{array}$$

$$f(x) = x^5 + x^4 - 2x + 3$$

Describe end behavior

$$x \rightarrow \infty, f(x) \rightarrow \infty$$

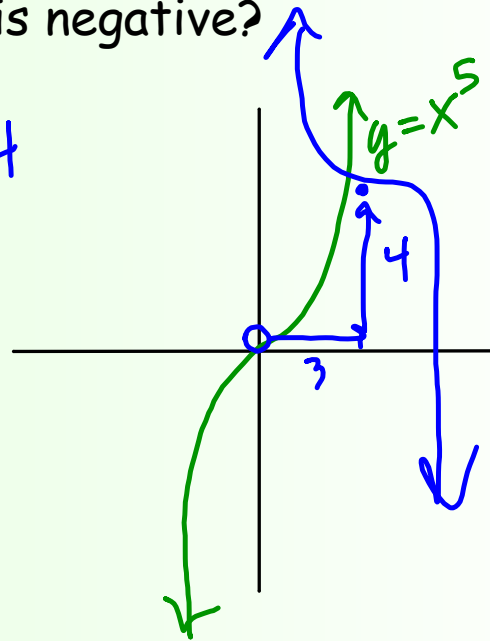
$$x \rightarrow -\infty, f(x) \rightarrow -\infty$$



What if leading coefficient is negative?

Try $f(x) = -2(x-3)^5 + 4$ ← up 4

R_x → vertical stretch
 Rt 3



Describe end behavior:

$$x \rightarrow \infty, f(x) \rightarrow -\infty$$

$$x \rightarrow -\infty, f(x) \rightarrow \infty$$

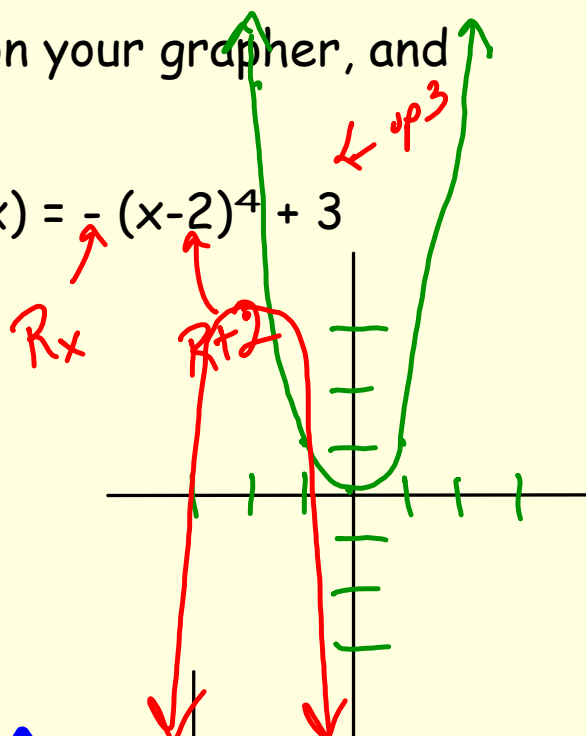
Even powered functions

Graph the functions below on your grapher, and describe end behavior.

$$f(x) = x^4$$

x	-2	-1	0	1	2
y	16	1	0	1	16

$$f(x) = -(x-2)^4 + 3$$



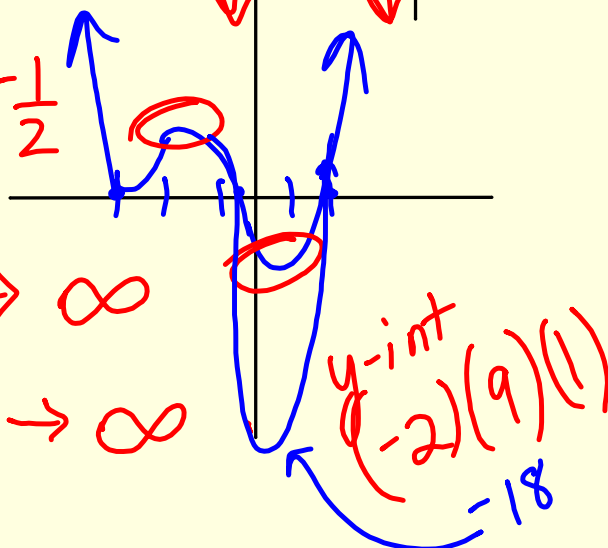
$$f(x) = (x-2)(x+3)^2(2x+1)^3$$

deg: 6

zeros: $x = 2, -3, -\frac{1}{2}$

$x \rightarrow \infty, f(x) \rightarrow \infty$

$x \rightarrow -\infty, f(x) \rightarrow \infty$



And what if leading coefficient were negative? Change one of the above and see what it looks like

Describe end behavior of

$$f(x) = ax^n + \dots$$

$$y = x^1$$

n is odd, a is positive

$$\text{as } x \Rightarrow \infty, f(x) \Rightarrow \infty$$

$$\text{as } x \Rightarrow -\infty, f(x) \Rightarrow -\infty$$

$$y = x^2$$

n is even, a is positive

$$\text{as } x \Rightarrow \infty, f(x) \Rightarrow \infty$$

$$\text{as } x \Rightarrow -\infty, f(x) \Rightarrow \infty$$

$$y = -x^1$$

n is odd, a is negative

$$\text{as } x \Rightarrow \infty, f(x) \Rightarrow -\infty$$

$$\text{as } x \Rightarrow -\infty, f(x) \Rightarrow \infty$$

$$y = -x^2$$

n is even, a is negative

$$\text{as } x \Rightarrow \infty, f(x) \Rightarrow -\infty$$

$$\text{as } x \Rightarrow -\infty, f(x) \Rightarrow -\infty$$

How many turning points, and how many zeros can an nth degree polynomial have?

deg: 3

Ex: $f(x) = (x-2)(x+5)(x-4)$ has 2 turning points ^{At most} and 3 zeros. ^{At most}

In the above example, find

the zeros

$$x = 2, -5, 4$$

the solution when $f(x) = 0$

$$0 = (x-2)(x+5)(x-4)$$

$$x = 2, -5, 4$$

the factors

$$(x-2)(x+5)(x-4)$$

the x-intercepts

$$(2, 0)(-5, 0)(4, 0)$$

Ex: $f(x) = (x-2)^2(x+5)^3(x-4)^5$ has 9 turning points at most and 10 zeros. At most

deg: 10

Zeros? $x = 2, -5, 4$

y-int? $(0,$

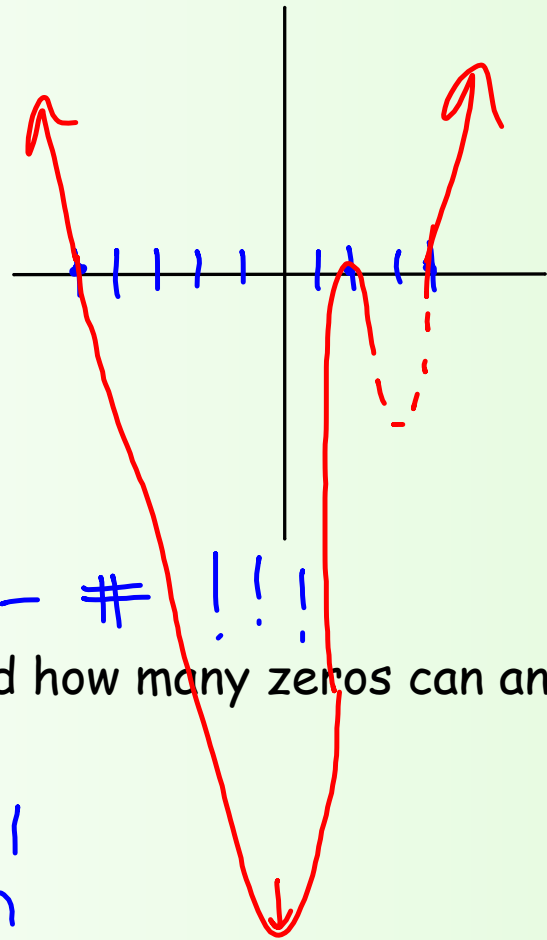
$$(-2)^2(5)^3(-4)^5$$

really big - # !!!

How many turning points, and how many zeros can an nth degree equation have?

$$\text{max \# turns} = n - 1$$

$$\text{max \# zeros} = n$$



Find all of the real zeros of $f(x) = x^4 + x^3 - 5x^2 - 5x$

$$\begin{aligned}
 f(x) &= x(x^3 + x^2 - 5x - 5) \\
 &= x(x^2(x+1) - 5(x+1)) \\
 f(x) &= x(x^2 - 5)(x+1) \\
 x &= 0, \pm\sqrt{5}, -1
 \end{aligned}$$

Find a polynomial with zeros at $-1/3, 4, 0$

$$\begin{aligned}
 x &= -\frac{1}{3} \quad x = 4 \quad x = 0 \\
 3x + 1 &= 0 \\
 f(x) &= (3x + 1)(x - 4)x
 \end{aligned}$$

HW: PC Book

p. 191 #1-8, 11-55 ☐,
and 43